CHAPTER 1
ELECTRIC CIRCUITS

1.1 Introduction

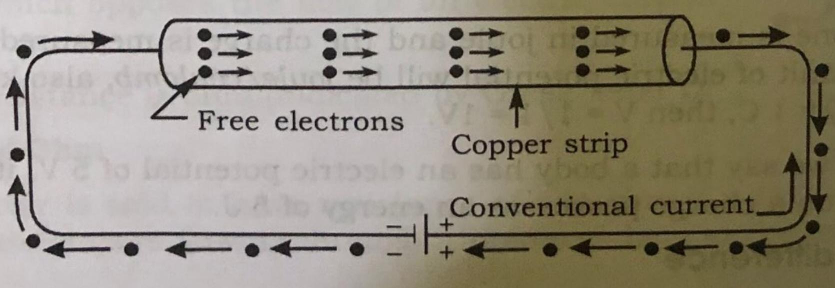
A dc circuit is a combination of circuit elements to form a closed path for the flow of dc current. A simple dc circuit consists of three basic elements : a battery, a lamp and connecting wires. In this circuit, battery is the voltage source and lamp is the load.

There are two types of elements found in electric circuits : passive elements and active elements. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors and inductors. Typical active elements include generators and batteries.

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them.

1.2 Electric Current

Flow of free electrons is called **electric current.** A copper strip has a large number of free electrons. When electric pressure or voltage is applied to it, free electrons, being negatively charged, will start moving towards the positive terminal round the circuit as shown in Fig.This directed flow of electrons is called electric current The actual direction of current (i.e., flow of electrons) is from the negative terminal to the positive terminal through the part of the circuit external to the cell. However, prior to the electron theory, it was assumed that current flowed from the positive terminal to the negative terminal of the cell via the circuit. This convention is so firmly established that it is still in use. This assumed current is now called *conventional current.*



Flow of electric current

The substances that have large number of free electrons will permit the flow of current easily. Such substances are called *conductors*, e.g., copper, zinc, silver, aluminium. On the other hand, atoms of some substances have valence electrons that are tightly held to their nuclei, i.e., they have few free electrons. Such substances will not permit the flow of electric current and are called *bad conductors* or *insulators*, e.g., glass, mica, porcelain.

The strength of electric current I is the rate of flow of electrons, i.e., charge flowing per second.

So, Current, $I = \frac{Q}{t}$

The charge Q is measured in coulomb and time t in second. Therefore, the unit of electric current will be *coulomb/sec,* also known as *ampere* (A). If $Q = 1$ C, $t = 1$ sec, then $I = 1/1 = 1$ A.

One ampere of current is said to flow through a wire if at any section one coulomb of charge flows in one second.

1.3 Electric Potential

A body is neutral under ordinary conditions, i.e., it contains the same number of protons and electrons so that the total positive charge of protons is exactly neutralized by the total negative charge of electrons. A body can be charged by removing the electrons from it or by supplying the electrons to it. Work is done in this process because electrons have to be removed or supplied against the opposing forces. This work done is stored in the body in the form of potential energy. The charged body has the capacity to do work by moving other charges either by attraction or by repulsion. This ability of the charged body to do work is called electric potential.

Thus, the capacity of a charged body to do work is called **electric potential.** The greater the capacity of a charged body to do work, the greater is its elec-tric potential. Obviously, the work done to charge a body to 1 C will be a measure of its electric potential, i.e.,

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

The work done is measured in joule and the charge is measured in coulomb. Therefore, the unit of electric potential will be *joule/coulomb,* also known as *volt* (V). If $W = 1$ J, $Q = 1$ C, then $V = 1/1 = 1$V.

Thus, when we say that a body has an electric potential of 5 V, it means that every coulomb on a charge possesses an energy of 5 J.

1.4 Potential difference

It is the difference in electrical potentials at any two given points.

It is also defined as the force which causes the electric current to flow in a closed circuit. It is measured in 'Volts'. Let 'V_a' and 'V_b' be the potentials of points 'a' and 'b' respectively with respect to ground. Then V_{ba} is the potential of point 'b' with respect to point 'a'. Thus,

$$V_{ba} = V_b - V_a$$

Electric current in any circuit always flows from higher potential to lower potential.

1.5 Electromotive Force [E.M.F]

The electromotive force is the pressure or voltage which causes an electric current to flow in a circuit.

Unit of e.m.f. is 'Volt'.

1.6 Power and Energy

Power of a machine is the rate of doing work. Usually unit of power in electricity is watt. Watt is the unit of power which is the product of voltage and current.

Let P = power, V = Voltage, I = Current, R = Resistance

Then $$P = VI = I^2R = \frac{V^2}{R}$$

In case of motors or engines, power is generally expressed in Horse Power (HP)

1 HP (Metric) = 735.5 watts

1 HP (British) = 746 watts

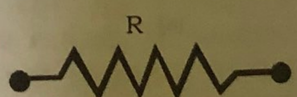
Energy of a body is the capacity for doing work. Electrical energy is generally expressed in joules and kilowatt hour.

Kilowatt hour is practical unit of electrical energy and is generally known as 1 unit of electrical energy.

Kilowatt hour = Power in kilowatt × Time in hours

1.7 Resistance

Resistance may be defined as that property of a substance which opposes the flow of an electric current through it.



Unit of resistance is ohm [Indicated by Greek letter Ω]

Definition of Ohm

A conductor is said to have resistance of one ohm if a D.C (Direct Current) current of one ampere flowing through it produces heat at the rate of one joule/second.

The circuit symbol for resistance appears in figure with the graphic abbreviation for resistance [R].

1.8 Ohm's Law

At constant temperature, the ratio of potential difference applied across the ends of a conductor and the current flowing through it remains constant.

If 'V' is the potential difference applied in volts and 'I' is the current in amperes, Ohm's law can be written as

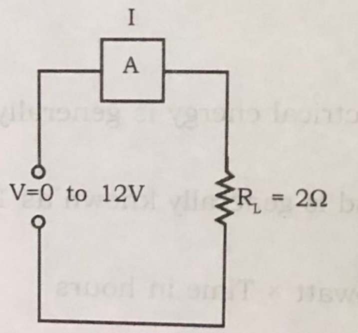
$$\frac{V}{I} = \text{a constant} = R$$

where 'R' is known as the resistance of the conductor.

1.9 Plotting Ohm's Law

The Ohm's law formula I = V/R states that V and I are directly proportional for any one value of R. This relation between V and I can be analyzed by using a fixed resistance of 2Ω for R_L, as in Fig. Then when V is varied, the meter shows I values directly proportional to V For instance. with 12 V, I equals 6 A; for 10 V, the current is 5 A; an 8-V potential difference produces 4 A.

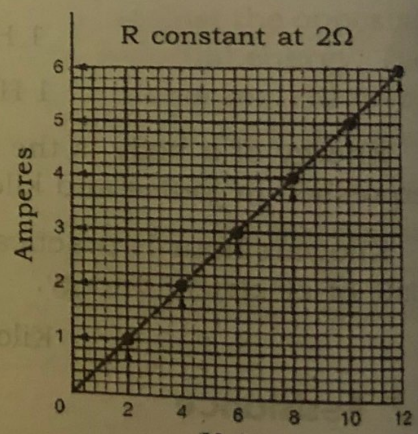
All the values of V and I are listed in the table and plotted in the graph . The I values are one-half the V values because R is 2Ω. However, I is zero with zero volts applied.



(a)

Volt V	Ohms Ω	Amperes A
0	2	0
2	2	1
4	2	2
6	2	3
8	2	4
10	2	5
12	2	6

(b)



(c)

Figure - Experiment to show that I increases in direct porportion to V with the same R. (a) Circuit with variable V but constant R. (b) Table of increase I for high V. (c) Graph of V and I values. This is a linear volt-ampere characteristics. It shows a direct proportion between V and I.

The voltage values for V are marked on the horizontal axis, called the x axis or abscissa. The current values I are on the vertical axis, called the y axis or ordinate.

Because the values for V and I depend on each other, they are variable factors. The independent variable here is V because we assign values of voltage and note the resulting current. Generally, the independent variable is plotted on the x axis, which is why the V values are shown here horizontally and the I values are on the ordinate.

The two scales need not be the same. The only requirement is that equal distances on each scale represent equal changes in magnitude. On the x axis here, 2-V steps are chosen, whereas the y axis has I-A scale divisions. The zero point at the origin is the reference.

The plotted points in the graph show the values in the table. For instance, the lowest point is 2V horizontally from the origin, and 1 A up. Similarly, the next point is at the intersection of the 4-V mark and the 2-A mark.

A line joining these plotted points includes all values of I, for any value of V, with R constant at 2Ω. This also applies to values not listed in the table. For instance, if we take the value of 7 V up to the straight line and over to the I axis, the graph shows 3.5 A for I.

Volt-Ampere Characteristic

The graph in Figure c is called the volt-ampere characteristic of R. It shows how much current the resistor allows for different voltages. Multiple and submultiple units of V and I can be used, though. For transistors, the units of I are often milliamperes or microamperes.

Linear Resistance

The straight-line (linear) graph in Figure shows that R is a linear resistor. A linear resistance has a constant value of ohms. Its R does not change with the applied voltage. Then V and I are directly proportional. Doubling the value of V from 4 to 8 V results in twice the current, from 2 to 4A. Similarly, three or four times the value of V will produce three or four times I, for a proportional increase in current.

Nonlinear Resistance

This type has a nonlinear volt-ampere characteristic. As an example, the resistance of the tungsten filament in a light bulb is nonlinear. The reason is that R increases with more current as the filament becomes hotter. Increasing the applied voltage does produce more current, but I does not increase in the same proportion as in the increase in V. Another example of a nonlinear resistor is a thermistor.

1.10 Resistivity

The resistance 'R' of a conductor depends on the following factors.

1. Material

The resistance of a conductor depends upon its material.

e.g., Silver is a better conductor than copper.

2. Length

The resistance 'R' of a conductor is directly proportional to the length 'l' of the conductor.

i.e., $R \propto l$

3. Cross-Sectional area

The resistance 'R' of a conductor is inversely proportional to cross-sectional area 'A' of the conductor.

i.e., $R \propto l/A$

4. Temperature

The resistance of metals and alloys increases with increase in temperature. But the resistance of Carbon, Silicon, and insulators decreases with rise in temperature.

From (2) and (3) we have

$$R \propto \frac{l}{A}$$

$$R = \frac{\rho \cdot l}{A}$$

where ρ (Greek letter rho) is the constant of proportionality and is called the resistivity or specific resistance of the material.

Since $$R = \frac{\rho \cdot l}{A}$$

Therefore $$\rho = \frac{R \cdot A}{l}$$

If $A = 1\ m^2$ and $l = 1$ m

Then $$\rho = R$$

Therefore, specific resistance is the resistance of a material having unit length and unit area of cross-section.

Unit of resistivity is Ohm-meter [Ω-m].

1.11 Conductors and Conductivity

Conductance is the property of a material by virtue of which it allows the passage of current through it easily. Conductance is the reciprocal of resistance.

i.e., Conductance, $$G = \frac{1}{R}$$

$$G = \frac{A}{\rho \cdot l} = \frac{\sigma A}{l}$$

where σ [Greek letter Sigma] is the specific conductance or conductivity of the material. Conductivity is the reciprocal of resistivity.

i.e., $$\sigma = \frac{1}{\rho}$$

Unit of conductance is mho.

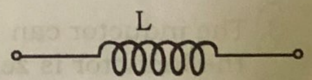
Unit of conductivity is mho per metre.

1.12 Inductors

A wire of certain length, when twisted into a coil becomes a basic inductor. If current is made to pass through an inductor, an electromagnetic field is formed. A change in the magnitude of the current changes the electromagnetic field. Increase in current expands the fields, and decrease in current reduces it. Therefore, a change in current produces change in the electromagnetic field, which induces a voltage across the coil according to Faraday's law of electromagnetic induction.

The unit of inductance is *henry,* denoted by *H.* By definition, the inductance is one henry when current through the coil, changing at the rate of one ampere per second, induces one volt across the coil. The symbol for inductance is shown in Figure.

The current-voltage relation is given by



$$v = L\frac{di}{dt}$$

where v is the voltage across inductor in voits, and i is the current through inductor in amps. We can rewrite the above equations as

$$di = \frac{1}{L} v dt$$

Integrating both sides, we get

$$\int_0^t di = \frac{1}{L}\int_0^t v dt$$

$$i(t) - i(0) = \frac{1}{L}\int_0^t v dt$$

$$i(t) = \frac{1}{L}\int_0^t v dt + i(0)$$

From the above equation we note that the current in an inductor is dependent upon the integral of the voltage across its terminals and the initial current in the coil, i(0)

The power absorbed by inductor is

$$P = vi = Li\frac{di}{dt} \text{ watts}$$

The energy stored by the inductor is

$$W = \int_0^t p\,dt$$

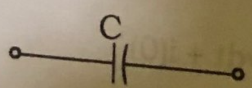
$$= \int_0^t Li\frac{di}{dt}\,dt = \frac{Li^2}{2}$$

From the above discussion, we can conclude the following.

1. The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.
2. A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor the current cannot change abruptly.
3. The inductor can store finite amount of energy, even if the voltage across the inductor is zero, and
4. A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

1.13 Capacitors

Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor. The conducting surfaces are called *electrodes*, and the insulating medium is called *dielectric*. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges, and is concentrated within the dielectric. The amount of charge per unit voltage that is capacitor can store is its capacitance, denoted by C. The unit of capacitance is *Farad* denoted by *F*. By definition, one Farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates. The symbol for capacitance is shown in Figure.



A capacitor is said to have greater capacitance if it can store more charge per unit voltage and the capacitance is given by

$$C = \frac{Q}{V}, \text{ or } C = \frac{q}{v}$$

(lower case letters stress instantaneous values)

We can write the above equation in terms of current as

$$i = C\frac{dv}{dt} \qquad \left(\because i = \frac{dq}{dt}\right)$$

where v is the voltage across capacitor, i is the current through it

$$dv = \frac{1}{C} i dt$$

Integrating both sides, we have

$$\int_0^t dv = \frac{1}{C}\int_0^t i dt$$

$$v(t) - v(0) = \frac{1}{C}\int_0^t i dt$$

$$v(t) = \frac{1}{C}\int_0^t i dt + v(0)$$

where v(0) indicates the initial voltage across the capacitor.

From the above equation, the voltage in a capacitor is dependent upon the integral of the current through it, and the initial voltage across it.

The power abosrbed by the capacitor is given by

$$p = vi = vC\frac{dv}{dt}$$

The energy stored by the capacitor is

$$W = \int_0^t p dt = \int_0^t vC\frac{dv}{dt} dt$$

$$W = \frac{1}{2} Cv^2$$

From the above discussion we can conclude the following

1. The current in a capacitor is zero if the voltage across it is constant;

that means, the capacitor acts as an open circuit to dc.

2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly.

3. The capacitor can store a finite amount of energy, even if the current through it is zero, and

4. A pure capacitor never dissipates energy, but only stores it; that is why it is called *non-dissipative passive element*. However, physical capacitors dissipate power due to internal resistance.

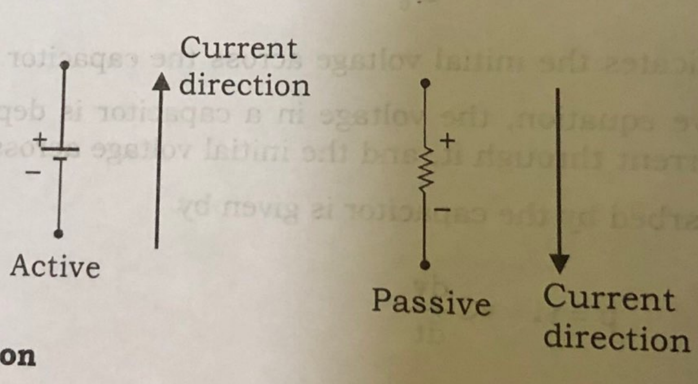
1.14 Active and passive sign convention

Active and passive sign convention is simply a methodology for designating positive and negative terminals on two-terminal circuit components for the purpose of easier circuit analysis.

Active Convention

Active sign convention is used for devices that deliver voltage to a circuit, for instance, batteries or discharging capacitors. For active components, the sign convention is very clear: the positive terminal is the terminal that deliver current (i.e., current flows out of it) and the negative terminal is the terminal that current flows back into. On most components these terminals are clearly marked and reversing them will provide a negative voltage.

The following figure illustrates the active and passive sign conventions.



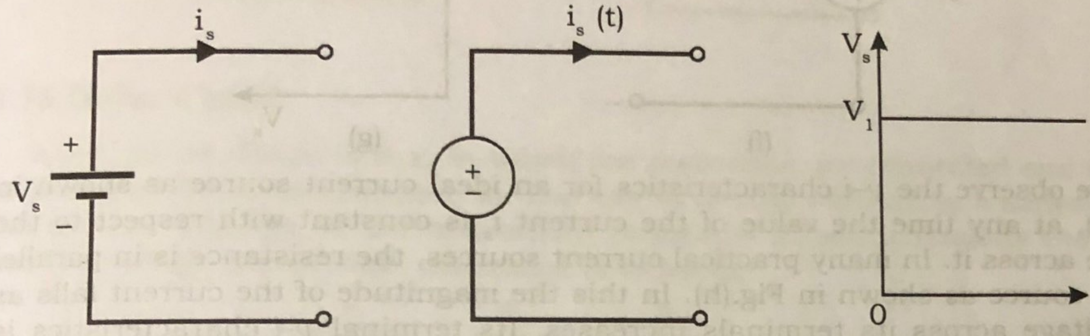
Passive convention

Resistors are the main components that follow passive sign convention. The point on a resistor where current flows into the component is considered its positive terminal, and the point where current flows out of the component is considered its negative terminal. If one was to physically examine a resistor, they would quickly see that there is no positive or negative terminals on the device. Passive sign convention does not depend on the device itself, simple on its orientation within the circuit. Mainly, the passive sign convention is simply a rule to designate terminals uniformly.

1.15 Energy Sources

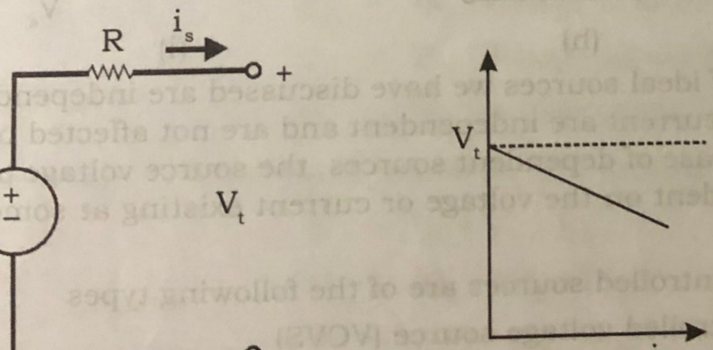
According to their terminal voltage-current characteristics, electrical energy sources are categorised into ideal voltage sources and ideal current sources. Further they can be divided into independent and dependent sources.

An ideal voltage source is a two-terminal element in which the voltage v_s is completely independent of the current i_s through its terminals. The representation of ideal constant voltage source is shown in Fig.(a).



(a) (b) (c)

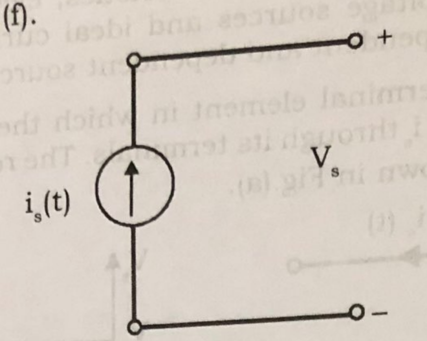
If we observe the v - i characteristics for an ideal voltage source as shown in Fig. (c) at any time, the value of the terminal voltage v_s is constant with respect to the value of current i_s. Whenever $v_s = 0$, the voltage source is the same that of a short circuit. Voltage sources need not have constant magnitude; in many cases the specified voltage may be time-dependent like a sinusoidal waveform. This may be represented as shown in Fig.(b). In many practical voltage sources, the internal resistance is represented in series with the source as shown in Fig.(d). In this, the voltage across the terminals falls as the current through it increases, as shown in Fig.(e).



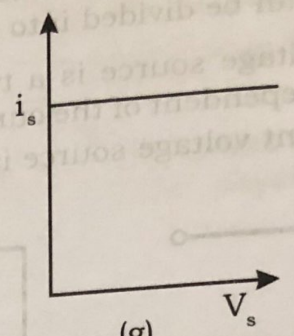
(d) (e)

The terminal voltage v_t depends on the source current as shown in Fig.(e), where $v_t = v_s - i_s R$.

An ideal constant current source is a two-terminal element in which the current i_s completely independent of the voltage v_s across its terminals. Like voltage sources we can have current sources of constant magnitude i_s or sources whose current varies with time $i_s(t)$. The representation of an ideal current source is shown in Fig.(f).

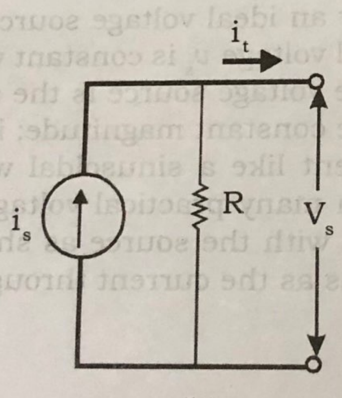


(f)

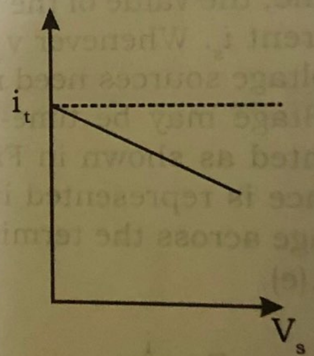


(g)

If we observe the v–i characteristics for an ideal current source as shown in Fig. (g), at any time the value of the current i_s is constant with respect to the voltage across it. In many practical current sources, the resistance is in parallel with a source as shown in Fig.(h). In this the magnitude of the current falls as the voltage across its terminals increases. Its terminal v-i characteristics is shown in Fig.(i). The terminal current is given by $i_t = i_s - (v_s/R)$. where R is the internal resistance of the ideal current source.



(h)



(i)

The two types of ideal sources we have discussed are independent sources for which voltage and current are independent and are not affected by other parts of the circuit. In the case of dependent sources, the source voltage or current is not fixed, but is dependent on the voltage or current existing at some other location in the circuit.

Dependent or controlled sources are of the following types

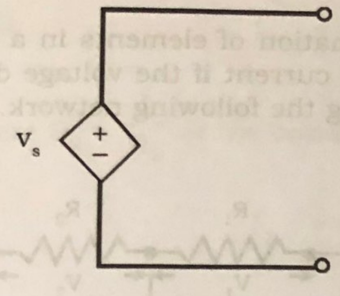
(i) voltage controlled voltage source (VCVS)

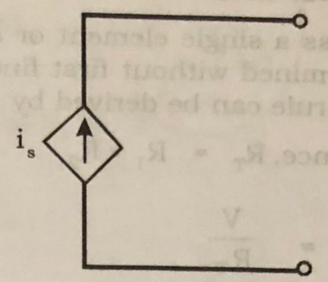
(ii) current controlled voltage source (CCVS)

(iii) voltage controlled current source (VCCS)

(iv) current controlled current source (CCCS)

These are represented in a circuit diagram by the symbol shown in Fig. These types of sources mainly occur in the analysis of equivalent circuits of transistors.





1.16 Series Circuit

A d.c. series circuit is that in which the resistance are connected end to end, so that they form only one path for the flow of electric current. In a series circuit the current through all the resistances connected in series remains the same, but the voltage across each resistance is different.

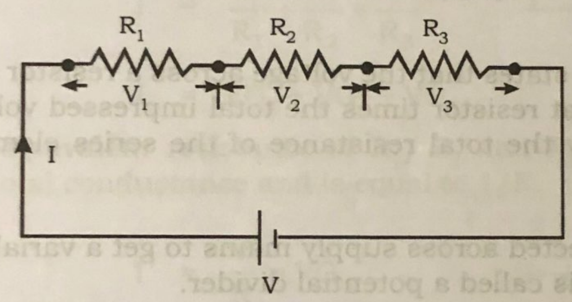


Figure shows three resistances R_1, R_2 and R_3 are connected in series to a voltage source V. Let I be the current flowing through the circuit.

Voltage drop across each resistance is given by Ohm's law.

$$V_1 = IR_1 \text{ [Voltage across } R_1]$$

$$V_2 = IR_2 \text{ [Voltage across } R_2]$$

$$V_3 = IR_3 \text{ [Voltage across } R_3]$$

The sum of all voltage drops is equal to the applied voltage 'V'.

i.e., $$V = V_1 + V_2 + V_3$$

$$V = IR_1 + IR_2 + IR_3$$

$$V = I(R_1 + R_2 + R_3)$$

But $$V = IR$$

Hence $$R = R_1 + R_2 + R_3$$

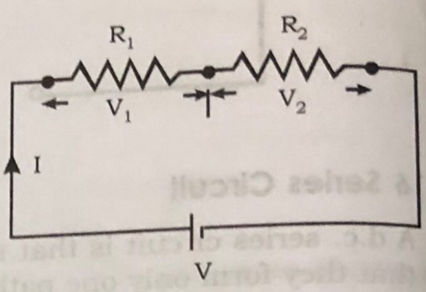
where R is the resistance of the total series circuit.

1.17 Voltage Divider Rule

The voltage across a single element or a combination of elements in a series circuit can be determined without first finding the current if the voltage divider rule is applied. The rule can be derived by analyzing the following network.

The total resistance, $R_T = R_1 + R_2$

$$I = \frac{V}{R_T}$$



$$V_1 = IR_1 = \left(\frac{V}{R_T}\right)R_1 = \frac{R_1 V}{R_T}$$

$$V_2 = IR_2 = \left(\frac{V}{R_T}\right)R_2 = \frac{R_2 V}{R_T}$$

The voltage divider rule states that the voltage across a resistor in series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

Potential Divider

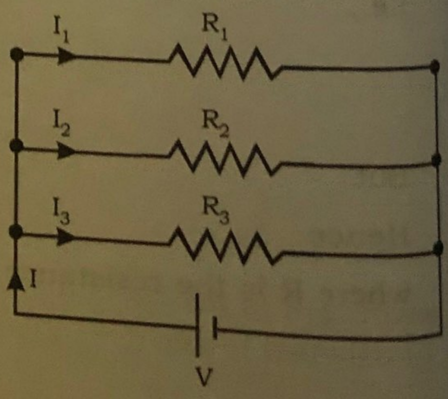
A high resistance connected across supply mains to get a variable voltage from a constant voltage supply is called a potential divider.

1.18 Parallel Circuit

A d.c. parallel circuit is that in which a number of resistances are connected in such a way that the starting ends of all the resistances are joined together and finishing ends of all the resistances are joined together. In parallel circuit the voltage across each resistance is the same but the currents through them differ from each other.

Figure shows three resistances R_1, R_2 and R_3 are connected in parallel and fed from a d.c. voltage source 'V'.

I_1 is the current through resistance R_1, I_2 is the current through resistances R_2 and I_3 is the current through resistance R_3.



Then $$I_1 = \frac{V}{R_1} = VG_1$$

where $G_1 = \frac{1}{R_1}$ is the conductance of R_1

$$I_2 = \frac{V}{R_2} = VG_2$$

where $G_2 = \frac{1}{R_2}$ is the conductance of R_2

$$I_3 = \frac{V}{R_3} = VG_3$$

where $G_3 = \frac{1}{R_3}$ is the conductance of R_3.

The total current I is equal to the sum of individual currents in each branch.

Then $$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{-----------(1)}$$

$$I = VG_1 + VG_2 + VG_3$$

If 'R' is the equivalent resistance of R_1, R_2 and R_3 in parallel and G is the corresponding total conductance and is equal to 1/R.

Hence, $$I = \frac{V}{R} = VG \quad \text{-----------(2)}$$

Comparing equation (1) and (2)

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$G = G_1 + G_2 + G_3$$

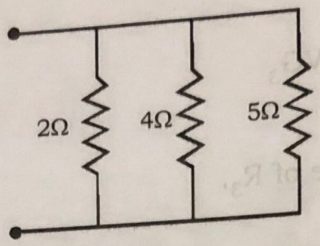
When a number of resistance are connected in parallel, the reciprocal of the total or equivalent resistance is given by the arithmetic sum of the reciprocals of their individual resistances.

When there are only two resistances connected in parallel then the total resistances R is given by the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Therefore
$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Example 1. Determine the total resistance for the following network.



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

$$= 0.5 + 0.25 + 0.2 = 0.95$$

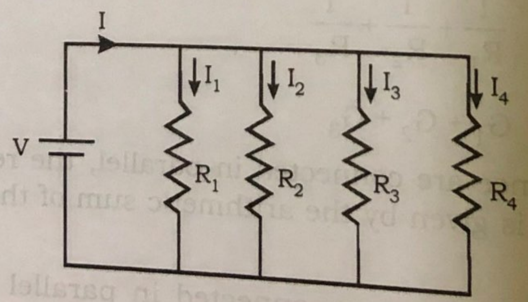
Therefore
$$R = \frac{1}{0.95} = 1.053\ \Omega$$

The above example demonstrate an interesting and useful characteristic of parallel resistors. The total resistance of parallel resistors is always less than the value of the smallest resistor.

1.19 Current Divider Rule

The current divider rule will determine how the current entering a set of parallel branches will split between the elements. For two elements of equal value, the current will divide equally. For elements with different values, the smaller the resistances, the greater the share of input current.

Consider the following network.



The input current I equals $\frac{V}{R}$ where R is the total resistance of the parallel branches.

Substituting $V = I_N R_N$ into the above equation, where I_N refers to the current through a parallel branch of resistance R_N.

$$I = \frac{V}{R} = \frac{I_N R_N}{R}$$

Therefore, $I_N = \frac{R}{R_N} I$ which is the general form for the current divider rule.

In words, the current through any parallel branch is equal to the product of the total resistance of the parallel branches and the input current divided by the resistance of the branch through which the current is to be determined.

For the current I_1,

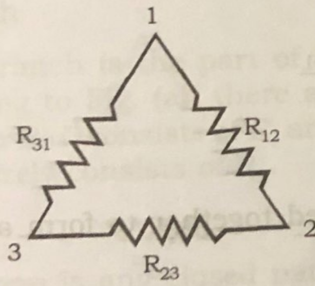
$$I_1 = \frac{R}{R_1} I$$

and for I_2

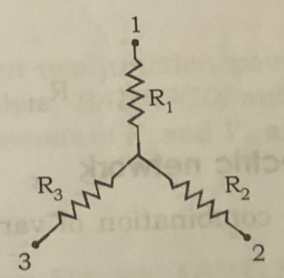
$$I_2 = \frac{R}{R_2} I$$

1.20 Delta-Star Transformation

The following figure shows three resistors R_{12}, R_{23} and R_{31} connected in the form of Delta. This delta network may be replaced by equivalent star network constituted by resistors R_1, R_2 and R_3 arranged in the form of a star.



Delta network



Equivalent star network

$$R_1 = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

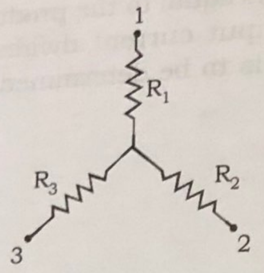
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

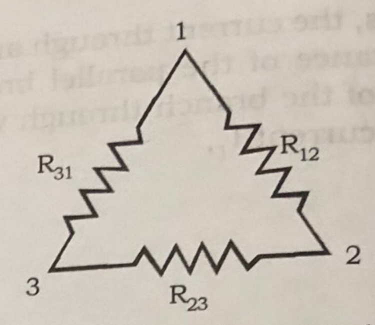
The rule for delta-star conversion may be stated as the product of adjacent two resistances of delta network divided by the sum of all the three resistances of the delta network.

1.21 Star-Delta Transformation

The following figure shows three resistors R_1, R_2 and R_3 connected in the form of star. This star network may be replaced by equivalent delta network constitute by resistors R_{12}, R_{23} and R_{31} arranged in the form of a delta.



Star network



Equivalent delta network

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

1.22 Electric network

It is a combination of various elements connected together to form a closed path.

Active elements

Active elements are voltage or current sources which are able to supply energy to the network.

Passive elements

Passive elements either absorb or store the energy from the sources. E.g. Resistors, Inductors and Capacitors.

Node

A **node** of network is an equipotential surface at which two or more circuit elements are joined.

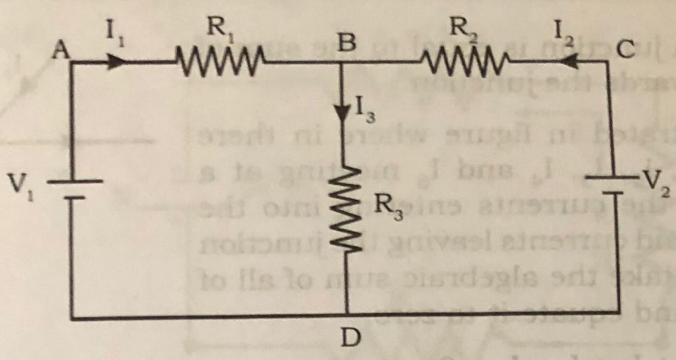


Fig. (a)

Thus, in Fig. (a) circuit elements R_1 and V_1 are joined at A and hence, A is the node. Similarly, B, C, and D are nodes.

Junction

A **junction** is that point in a network where three or more circuit elements are joined. In Fig.(a), there are only two junction points, B and D. That B is a junction is clear from the fact that three circuit elements R_1, R_2, and R_3 are joined at it. Similarly, point D is a junction because it joins three circuit elements R_3, V_1 and V_2. All the junctions are the nodes but all the nodes are not junctions.

Branch

A branch is the part of a network lying between two junction points. Thus, referring to Fig. (a), there are total of three branches, *BAD*, *BCD*, and *BD*. The branch *BAD* consists of R_1 and V_1 the branch *BCD* consists of R_2 and V_2, and branch *BD* merely consists of R_3.

Loop

A **loop** is any closed path of a network. Thus, in Fig. (a), *ABDA*, *BCDB*, and *ABCDA* are the loops.

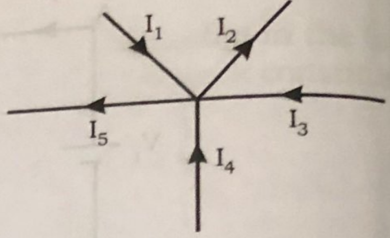
Mesh

A **mesh** is the most elementary form of a loop and cannot be further divided into other loops. In fig. (a), both loops *ABDA* and *BCDB* are meshes because they cannot be further divided into other loops. However, the loop *ABCDA* cannot be called a mesh because it encloses two loops *ABDA* and *BCDB*. All meshes are loops but all loops are not meshes.

1.23 Kirchhoff's laws

Kirchhoff's law enable us to calculate the current flowing in various branches of a network as well as the voltages at various points of the network. The two laws are given by Kirchhoff's are the current law and the voltage law.

1. Kirchhoff's current law (KCL). It states that the algebraic sum of currents meeting at a junction in a network is zero. In other words, the sum of the currents flowing away from a junction is equal to the sum of currents flowing towards the junction.



This law is illustrated in figure where in there are five currents I_1, I_2, I_3, I_4 and I_5 meeting at a junction. Assuming the currents entering into the junction as positive and currents leaving the junction as negative. We can take the algebraic sum of all of these five currents and equate it to zero.

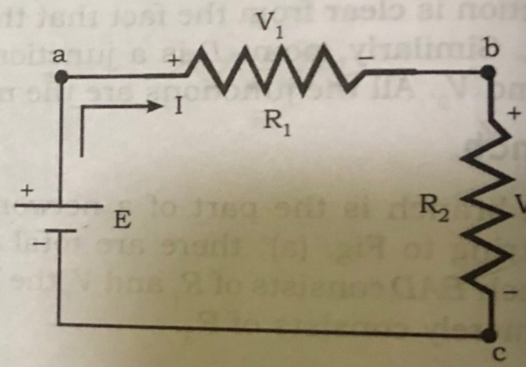
$$I_1 - I_2 + I_3 + I_4 - I_5 = 0$$

Also we can write KCL as the sum of currents flowing towards the junction equal to the sum of currents flowing away from the junction.

$$I_1 + I_3 + I_4 = I_2 + I_5$$

2. Kirchhoff's Voltage Law (K.V.L.). For any closed path in a network, Kirchhoff's voltage law states that the algebraic sum of the voltage is zero.

In other words, KVL states that the algebraic sum of the potential rises and drops around a closed path is zero.



In the above figure, we can trace a continuous path that leaves point 'a' through R_1 and returns through E without leaving the circuit. Therefore, 'abca' is a closed loop. A plus sign is assigned to a potential rise (– to +) and a minus sign to a potential drop (+ to –). If we follow the current in the figure from point 'a' we first encounter a potential drop V_1 (+ to –) across R_1 and then another potential drop V_2 across R_2. Continuing through the voltage source, we have a potential rise E (– to +) before returning to point 'a'.

According to Kirchhoff's law,

$$+E - V_1 - V_2 = 0$$

i.e., $$E = V_1 + V_2$$

The potential impressed on the circuit by the battery is equal to the potential drops within the circuit.

In the above circuit $V_1 = IR_1$ and $V_2 = IR_2$. Then E can be written as

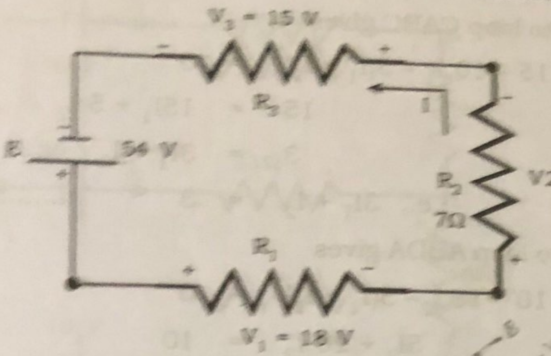
$$E = IR_1 + IR_2$$

***Example. 1.* For the following circuit**

(a) Determine V_2 using Kirchhoff's Voltage law

(b) Determine I

(c) Find R_1 and R_3



(a) Applying Kirchhoff's voltage law

$$-E + V_3 + V_2 + V_1 = 0$$

$$E = V_1 + V_2 + V_3$$

$$V_2 = E - V_1 - V_3 = 54 - 18 - 15$$

$$V_2 = 21\ V$$

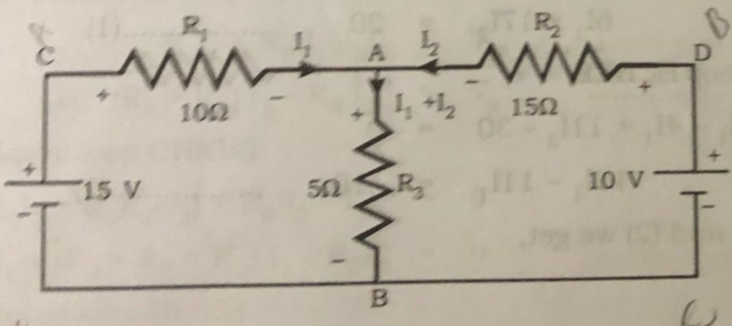
(b) $$V_2 = IR_2$$

$$\therefore I = \frac{V_2}{R_2} = \frac{21}{7} = 3A$$

(c) $$R_1 = \frac{V_1}{I} = \frac{18}{3} = 6\ \Omega$$

$$R_3 = \frac{V_3}{I} = \frac{15}{3} = 5\Omega$$

2. For the following circuit, find the currents in resistors R_1, R_2 and R_3.



Applying KVL to loop CABC gives

$$15 - 10\,I_1 - 5(I_1 + I_2) = 0$$

$$15 = 15I_1 + 5\,I_2$$

$$3 = 3I_1 + I_2$$

$$\text{i.e.,}\quad 3I_1 + I_2 = 3 \qquad \text{.......................(1)}$$

Applying KVL to loop ABDA gives

$$10 - 15\,I_2 - 5(I_1 + I_2) = 0$$

$$5I_1 + 20\,I_2 = 10$$

$$I_1 + 4I_2 = 2 \qquad \text{.......................(2)}$$

On solving Eqns. (1) and (2) we get,

$$I_1 = \frac{10}{11}\text{A},\ I_2 = \frac{3}{11}\text{A},\ I_1 + I_2 = \frac{13}{11}\text{A}$$

Current through resistor $R_1 = \frac{10}{11}$ A

Current through resistor $R_2 = \frac{3}{11}$ A

Current through resistor $R_3 = \frac{13}{11}$ A

3. In the circuit shown in figure, Determine the direction and value of current in each of the batteries.

Applying Kirchhoff's Voltage law to loop ABEFA,

$$30 - 11I_2 - 6(I_1 + I_2) - 10 = 0$$

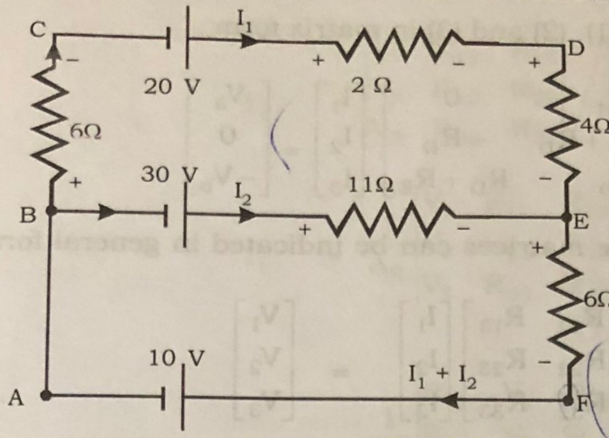
$$6I_1 + 17I_2 = 20 \qquad \text{................(1)}$$

Applying KVL to loop BCDEB

$$-6I_1 + 20 - 2I_1 - 4I_1 + 11I_2 - 30 = 0$$

$$12I_1 - 11I_1 = -10 \qquad \text{................(2)}$$

Solving Eqns. (1) and (2) we get,

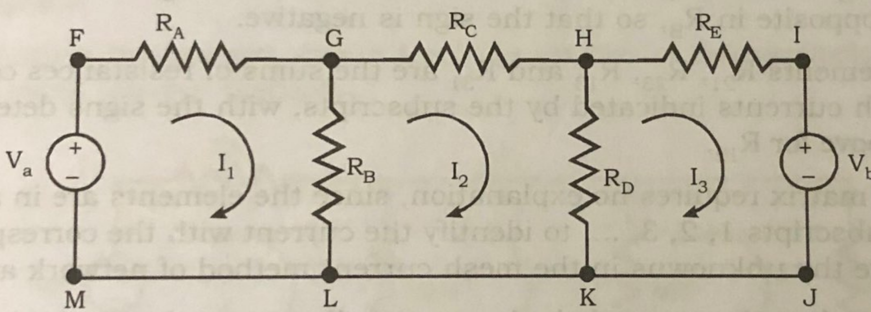


Current through 20 V battery = 0.185 A

Current through 30 V battery = 1.11 A

Current through 10 V battery = 1.295 A

Matrices and Mesh Currents



When KVL is applied to the above network the following three equations are obtained. For the closed loop FGLMF

$$R_A I_1 + R_B(I_1 - I_2) = V_a$$

$$\text{i.e., } (R_A + R_B)\, I_1 - R_B\, I_2 = V_a \quad \ldots\ldots\ldots\ldots\ldots (1)$$

For the closed loop GHKLG

$$R_C I_2 + R_D(I_2 - I_3) + R_B(I_2 - I_1) = 0$$

$$\text{i.e., } -R_B I_1 + (R_B + R_C + R_D)\, I_2 - R_D I_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

For the closed loop HIJKH

$$R_E I_3 + R_D(I_3 - I_2) = -V_b$$

$$\text{i.e., } -R_D I_2 + (R_D + R_E) I_3 = -V_b \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Placing the Eqns. (1), (2) and (3) in matrix form

$$\begin{bmatrix} R_A + R_B & -R_B & 0 \\ -R_B & R_B + R_C + R_D & -R_D \\ 0 & -R_D & R_D + R_E \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -V_b \end{bmatrix}$$

The elements of the matrices can be indicated in general form as follows

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Now element R_{11} (row 1, column 1) is the sum of all resistances through whic mesh current I_1 passes. In the figure, this is $R_A + R_B$. Similarly, elements R_{22} an R_{33} are the sums of all resistances through which I_2 and I_3, respectively, pass.

Element R_{12} (row 1, column 2) is the sum of all resistances through whic mesh currents I_1 and I_2 pass. The sign of R_{12} is '+' if the two currents are in th same direction through each resistance, and '–' if they are in opposite directions. In the figure, R_B is the only resistance common to I_1 and I_2; and the curren directions are opposite in R_B, so that the sign is negative.

Similarly, elements R_{21}, R_{23}, R_{13} and R_{31} are the sums of resistances commo to the two mesh currents indicated by the subscripts, with the signs determine as described above for R_{12}.

The current matrix requires no explanation, since the elements are in a singl column with subscripts 1, 2, 3, ... to identify the current with the correspondin mesh. These are the unknowns in the mesh current method of network analysis.

Element V_1 in the voltage matrix is the sum of all source voltages driving mes current I_1. A voltage is counted positive in the sum if I_1 passes from '–' to the '+' terminal of the source; otherwise it is counted negative. In other words, a voltag is positive if the source drives in the direction of the mesh current.

1.24 Determinants and the Maxwell's Mesh current method

The matrix equation arising from the mesh current method may be solved b various techniques. One of these, the method of determinants (Cramer's rule will be presented here.

The unknown current I_1 is obtained as the ratio of two determinants. Th denominator determinant has the elements of the resistance matrix. This ma be referred to as the determinant of the coefficients and given the symbol Δ_R.

The numerator determinant has the same elements as Δ_R except in the firs columns, where the elements of the voltage matrix replace those of the determinat of the coefficients. Thus

$$I_1 = \begin{vmatrix} V_1 & R_{12} & R_{13} \\ V_2 & R_{22} & R_{23} \\ V_3 & R_{32} & R_{33} \end{vmatrix} \Bigg/ \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$$I_1 = \frac{1}{\Delta_R} \begin{vmatrix} V_1 & R_{12} & R_{13} \\ V_2 & R_{22} & R_{23} \\ V_3 & R_{32} & R_{33} \end{vmatrix}$$

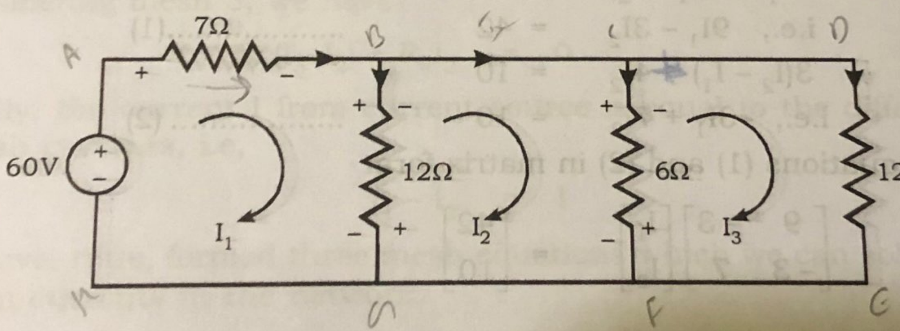
Similarly,

$$I_2 = \frac{1}{\Delta_R} \begin{vmatrix} R_{11} & V_1 & R_{13} \\ R_{21} & V_2 & R_{23} \\ R_{31} & V_3 & R_{33} \end{vmatrix}$$

$$I_3 = \frac{1}{\Delta_R} \begin{vmatrix} R_{11} & R_{12} & V_1 \\ R_{21} & R_{22} & V_2 \\ R_{31} & R_{32} & V_3 \end{vmatrix}$$

Example 1

Calculate the current drawn from the source in the network shown in figure using mesh current method.



Applying KVL to each mesh,

$$60 - 7\,I_1 - 12\,(I_1 - I_2) = 0$$

$$19\,I_1 - 12\,I_2 = 60 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$-6(I_2 - I_3) - 12\,(I_2 - I_1) = 0$$

$$12\,I_1 - 18\,I_2 + 6\,I_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$-12I_3 - 6(I_3 - I_2) = 0$$

$$0 + 6I_2 - 18I_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Putting the equations in matrix form,

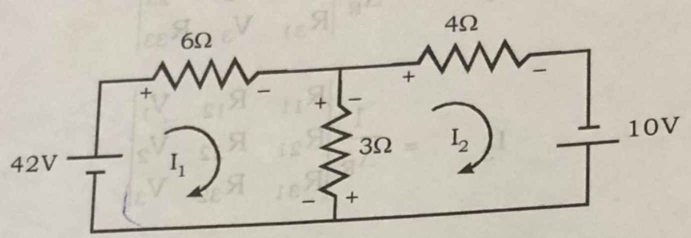
$$\begin{bmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

Using Creamer's rule, we get

$$I_1 = \begin{vmatrix} 60 & -12 & 0 \\ 0 & 18 & -6 \\ 0 & -6 & 18 \end{vmatrix} \div \begin{vmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{vmatrix} = 17280 \div 2880 = \mathbf{6A}$$

Example 2

Write the mesh equations for the following network and find the current in the 4Ω resistor?



Applying KVL to each mesh results in

$$6I_1 + 3(I_1 - I_2) = 42$$

$$\text{i.e.,} \quad 9I_1 - 3I_2 = 42 \quad \text{..................(1)}$$

$$3(I_2 - I_1) + 4I_2 = 10$$

$$\text{i.e.,} \quad -3I_1 + 7I_2 = 10 \quad \text{..................(2)}$$

Pulling the equations (1) and (2) in matrix form

$$\begin{bmatrix} 9 & -3 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix}$$

$$I_1 = \begin{vmatrix} 42 & -3 \\ 10 & 7 \end{vmatrix} \div \begin{vmatrix} 9 & -3 \\ -3 & 7 \end{vmatrix} = 324 \div 54 = 6A$$

$$I_2 = \begin{vmatrix} 9 & 42 \\ -3 & 10 \end{vmatrix} \div \begin{vmatrix} 9 & -3 \\ -3 & 7 \end{vmatrix} = 216 \div 54 = 4A$$

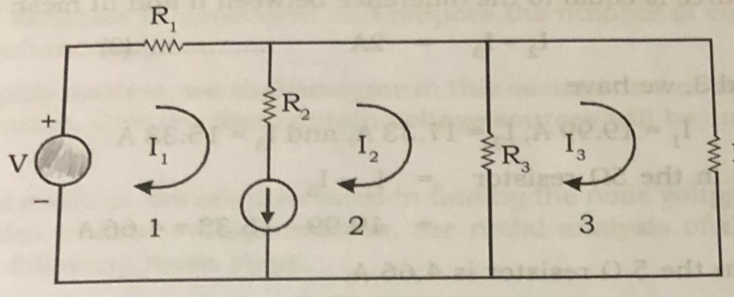
Current through the 4Ω resistor $= 4$ A

Current through the 3Ω resistor $= I_1 - I_2 = 6 - 4 = 2$ A

1.25 Supermesh Analysis

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis straight forward because first we should assume as unknown voltage across the current source, writing mesh equations as before, and then relate the source current to the assigned mesh currents. This is generally a difficult approach. One way to overcome this difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in fig.

Here, the current source I is in the common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2.



$$R_1I_1 + R_3(I_2 - I_3) = V$$

or

$$R_1I_1 + R_3I_2 - R_4I_3 = V$$

Considering mesh 3, we have

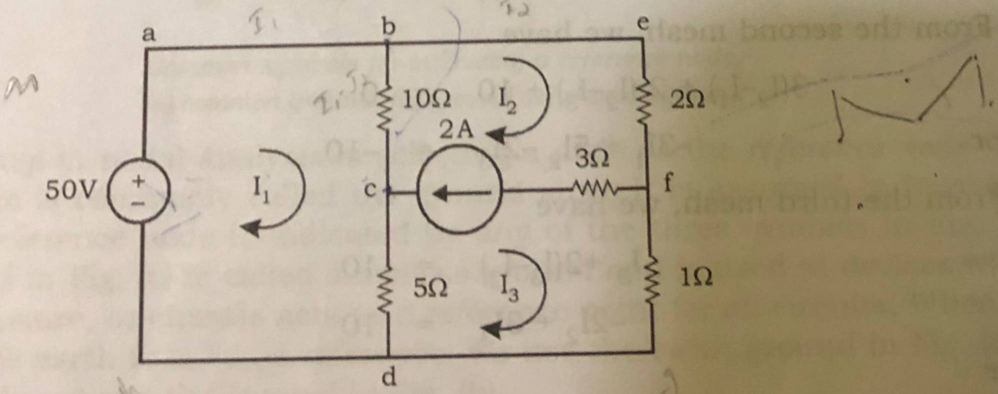
$$R_3(I_3 - I_2) + R_4I_3 = 0$$

Finally, the current I from current source is equal to the difference between two mesh currents, i.e,

$$I_1 - I_2 = I$$

We have, thus, formed three mesh equations which we can solve for the three unknown currents in the network.

Example 1. Determine the current in the 5Ω resistor in the network given in Fig.



50 = 10(I1 - I2) - 5(I1 - I3)

Solution. From the first mesh, i.e., abcda, we have

$$50 = 10(I_1 - I_2) + 5(I_1 - I_3)$$

or $$15I_1 - 10\,I_2 - 5I_3 = 50 \quad \text{.........(1)}$$

From the second and third meshes, we can form a supermesh

$$10(I_2 - I_1) + 2I_2 + I_3 + 5(I_3 - I_1) = 0$$

or $$-15I_1 + 12\,I_2 + 6I_3 = 0 \quad \text{.........(2)}$$

The current source is equal to the difference between II and III mesh currents

i.e, $$I_2 - I_3 = 2\text{A} \quad \text{.........(3)}$$

Solving 1, 2 and 3, we have

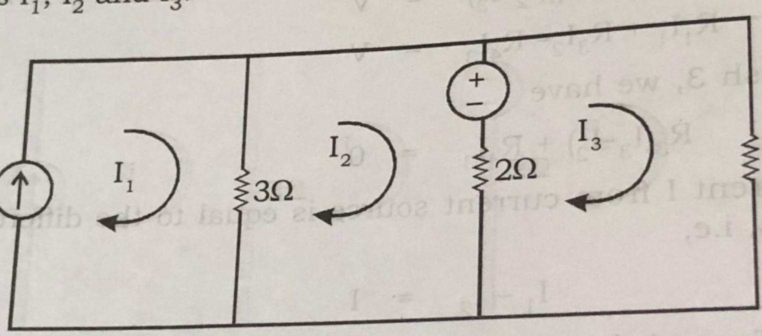
$$I_1 = 19.99 \text{ A}, I_2 = 17.33 \text{ A, and } I_3 = 15.33 \text{ A}$$

$$\text{The current in the } 5\Omega \text{ resistor} = I_1 - I_3$$

$$= 19.99 - 15.33 = 4.66 \text{ A}$$

∴ The current in the 5 Ω resistor is 4.66 A.

Example 2. Write the mesh equations for the circuit shown in Fig. and determine the currents I_1, I_2 and I_3.



Solution. In Fig., the current source lies on the perimeter of the circuit, and the first mesh is ignored. Kirchhoff's voltage law is applied only for second and third meshes.

From the second mesh, we have

$$3(I_2 - I_1) + 2\,(I_2 - I_3) + 10 = 0$$

or $$-3I_1 + 5I_2 - 2I_3 = -10$$

From the third mesh, we have

$$I_3 + 2(I_3 - I_2) = 10$$

$$-2I_2 + 3I_3 = 10$$

From the first mesh, $I_1 = 10\text{ A}$

From the above three equations, we get

$$I_1 = 10\text{A},\ I_2 = 7.27\text{ A},\ I_3 = 8.18\text{ A}$$

1.26 Nodal Analysis

Nodal analysis is also known as the node-voltage method

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

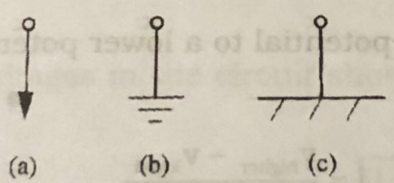
To simplify matters, we shall assume in this section that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed in the next section.

In nodal analysis, we are interested in finding the node voltages. Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

Steps to Determine Node Voltages

1. Select a node as the reference node. Assign voltages v_1, v_2, ...v_{n-1} to the remaining n −1 nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the n - 1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

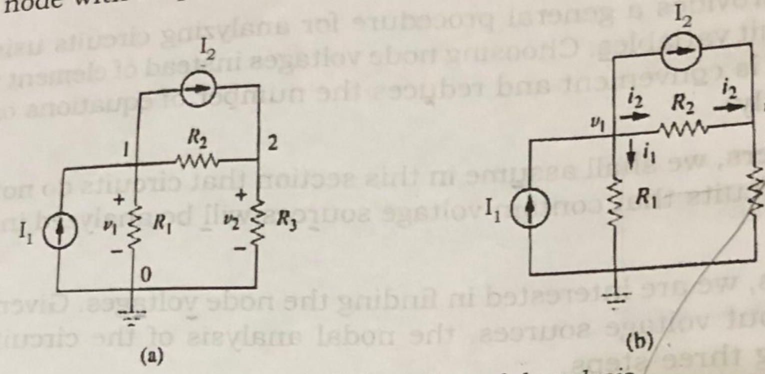
We shall now explain and apply these three steps.



Common symbols for indicating a reference node,
(a) common ground, (b) ground, (c) chassis ground

The first step in nodal analysis is selecting a node as the *reference node*. The reference node is commonly called the ground since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in Fig. The type of ground in Fig. (c) is called a chassis ground and is used in devices where the case, enclosure, or chassis acts as a reference point for all circuits. When the potential of the earth is used as reference, we use the earth ground in Fig. (a) or (b). We shall always use the symbol in Fig. (b).

Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit in Fig. (a). Node 0 is the reference node ($v = 0$), while nodes 1 and 2 are assigned voltages v_1 and v_2 respectively. Keep in mind that the node voltages are defined with respect to the reference node. As illustrated in Fig. (a), each node voltage is the voltage rise from the reference node to the corresponding nonreference node or simply the voltage of that node with respect to the reference node.



Typical circuit for nodal analysis

As the second step, we apply KCL to each nonreference node in the circuit. To avoid putting too much information on the same circuit, the circuit in Fig.(a) is redrawn in Fig. (b), where we now add i_1, i_2, and i_3 as the currents through resistors R_1, R_2, and R_3, respectively. At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2 \quad \ldots\ldots\ldots\ldots (1)$$

At node 2,

$$I_2 + i_2 = i_3 \quad \ldots\ldots\ldots\ldots (2)$$

We now apply Ohm's law to express the unknown currents i_1, i_2, and i_3 in terms of node voltages. The key idea to bear in mind is that, since resistance is a passive element, by the passive sign convention, current must always flow from a higher potential to a lower potential.

Current flows from a higher potential to a lower potential in a resistor. We can express this principle as

$$i = \frac{v_{higher} - v_{lower}}{R}$$

We obtain from Fig. (b),

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or } i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or } i_2 = G_2(v_1 - v_2) \ldots\ldots\ (3)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$

Substituting Eq. (3) in Eqs. (1) and (2) results, respectively,

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \quad \text{.........} \quad (4)$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \quad \text{.........} \quad (5)$$

In terms of the conductances, Eqs. (4) and (5) become

$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2) \quad \text{..........} \quad (6)$$

$$I_2 + G_2(v_1 - v_2) = G_3 v_2 \quad \text{...........} \quad (7)$$

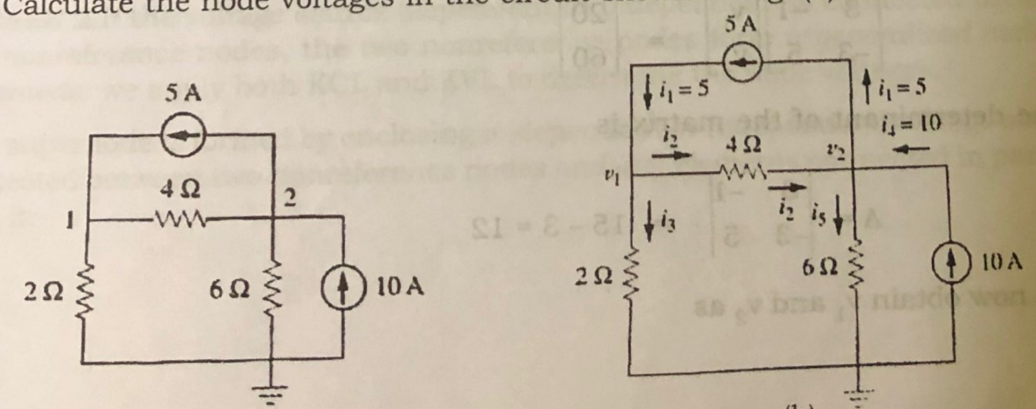
The third step in nodal analysis is to solve for the node voltages. If we apply KCL to n −1 nonreference nodes, we obtain n−1simultaneous equations such as Eqs. (4) and (5) or (6) and (7). For the circuit of Fig. (a), (b) we solve Eqs. (4) and (5) or (6) and (7) to obtain the node voltages v_1 and v_2 using any standard method, such as the substitution method, the elimination method, Cramer's rule, or matrix inversion. To use either of the last two methods, one must cast the simultaneous equations in matrix form. For example, Eqs. (6) and (7) can be cast in matrix form as

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

which can be solved to get v_1 and v_2.

Example

Calculate the node voltages in the circuit shown in Fig. (a).



(a) (b)

Fig. *(a) original circuit (b) circuit for analysis*

Solution:

Consider Fig. (b), where the circuit in Fig. (a) has been prepared for nodal analysis. Notice how the currents are selected for the application of KCL. Except for the branches with current sources, the labeling of the currents is arbitrary but consistent. (By consistent, we mean that if, for example, we assume that i_2 enters the 4Ω resistor from the left-hand side, i_2 must leave the resistor from the right-hand side.) The reference node is selected, and the node voltages v_1 and v_2 are now to be determined.

At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term in the last equation by 4, we obtain

$$20 = v_1 - v_2 + 2v_1$$

$$3v_1 - v_2 = 20 \ \ldots\ldots \quad (1)$$

At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Multiplying each term by 12 results in

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

$$-3v_1 + 5v_2 = 60 \ \ldots\ldots\ldots \ (2)$$

Now we have two simultaneous Eqs. (1) and (2). We can solve the equations using any method and obtain the values of v_1 and v_2.

To use Cramer's rule, we need to put Eqs. (1) and (2) in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

We now obtain v_1 and v_2 as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100+60}{12} = 13.333 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180+60}{12} = 20 \text{ V}$$

If we need the currents, we can easily calculate them from the values of the nodal voltages

$$i_1 = 5 \text{ A}, \quad i_2 = \frac{v_1 - v_2}{4} = -1.6668 \text{ A}, \quad i_3 = \frac{v_1}{2} = 6.666 \text{ A}$$

$$i_4 = 10 \text{ A}, \quad i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

The fact that i_2 is negative shows that the current flows in the direction opposite to the one assumed.

Nodal Analysis with Voltage Sources

We now consider how voltage sources affect nodal analysis. We use the circuit in Fig. for illustration. Consider the following two possibilities.

CASE 1 If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. 3.7, for example,

$$v_1 = 10 \text{ V} \quad \text{.......... (1)}$$

Thus our analysis is somewhat simplified by this knowledge of the voltage at this node.

CASE 2 If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a *generalized node* or *supernode:* we apply both KCL and KVL to determine the node voltages.

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

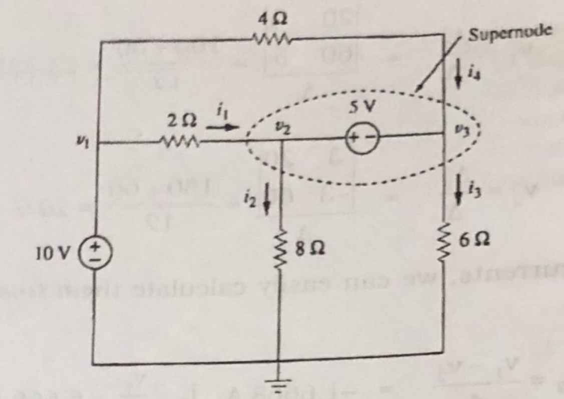


Fig. *A circuit with a supernode*

In Fig. , nodes 2 and 3 form a supernode. (We could have more than two nod forming a single supernode). We analyze a circuit with supernodes using the sa three steps mentioned in the previous section except that the supernodes a treated differently. Because an essential component of nodal analysis is applyi KCL, which requires knowing the current through each element. There is no w of knowing the current through a voltage source in advance. However, KCL mu be satisfied at a supernode like any other node. Hence, at the supernode in Fi

$$i_1 + i_4 = i_2 + i_3$$

or $$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6} \quad \text{..........} (2)$$

To apply Kirchhoff's voltage law to the supernode in Fig. , we redraw the circ as shown in Fig. Going around the loop in the clockwise direction gives

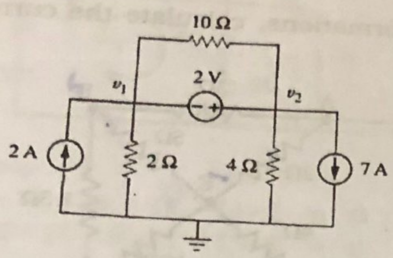
$$-v_2 + 5 + v_3 = 0 \quad \Rightarrow v_2 - v_3 = 5 \quad \text{...........} (3)$$

From Eqs. (1), (2), and (3), we obtain the node voltages. Note the followi properties of a supernode:

1. The voltage source inside the supernode provides a constraint equati needed to solve for the node voltages.
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KCL and KVL.

Example

For the circuit shown in Fig. find the node voltages



Solution:

The supernode contains the 2 V source, nodes 1 and 2, and the 10 Ω resistor. Applying KCL to the supernode as shown in Fig. gives

$$2 = i_1 + i_2 + 7$$

Expressing i_1 and i_2 in terms of the node voltages.

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Rightarrow 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \text{ .... } (1)$$

To get the relationship between v_1 and v_2, we apply KVL to the circuit in Fig.(b) Going around the loop, we obtain

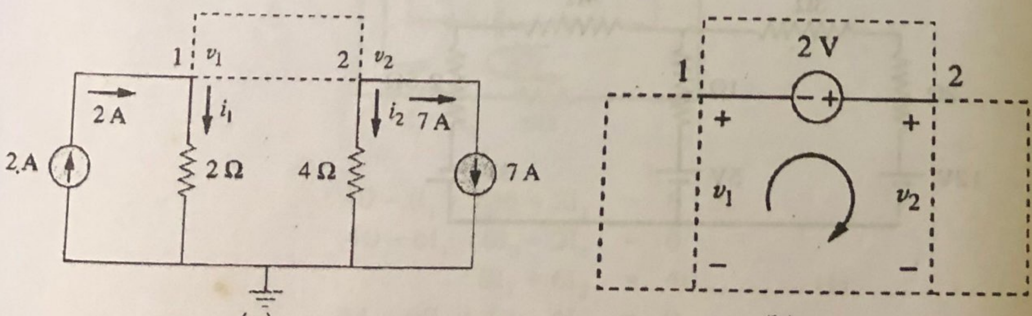
$$-v_1 - 2 + v_2 = 0 \quad \Rightarrow v_2 = v_1 + 2 \text{ .... } (2)$$

From Eqs. (1) and (2), we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

$$3v_1 = -22 \quad \Rightarrow v_1 = -7.333 \text{ V}$$

and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10 Ω resistor does not make any differene because it is connected across the supernode.

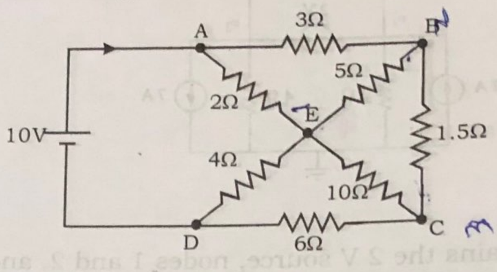


(a) (b)

Fig. *Applying (a) KCL to the supernode, (b) KVL to the loop*

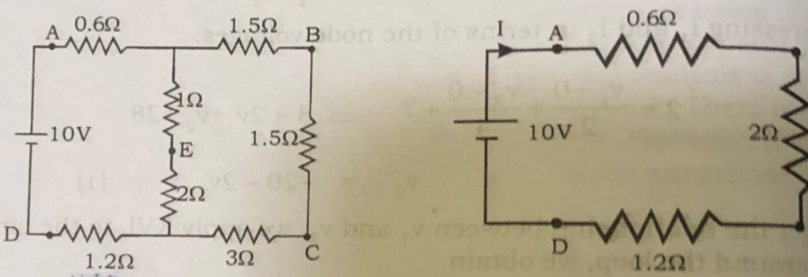
PROBLEMS

1. Using delta-star transformations, calculate the current I drawn from the battery in the following.



Solution

Delta ABE and DCE are converted into corresponding star networks as shown in figure.

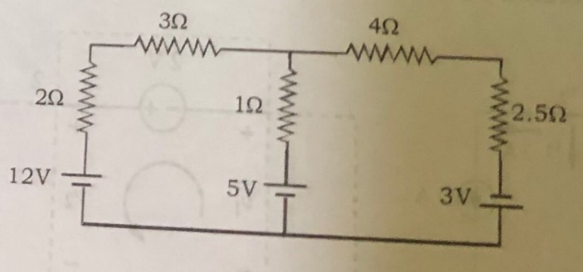


Current I = 10/3.8 = 2.63 A

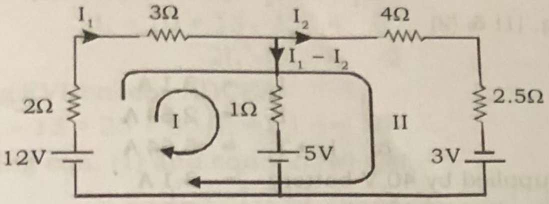
2. Using Kirchoff's laws determine

(i) Current in the 2.5 Ω resistor.

(ii) Voltage across 1Ω resistor.



Ans.



Applying KVL on loop I,

$$12 - 2I_1 - 3I_1 - 1(I_1 - I_2) - 5 = 0$$
$$12 - 2I_1 - 3I_1 - I_1 + I_2 - 5 = 0$$
$$7 - 6I_1 + I_2 = 0$$
$$6I_1 - I_2 = 7 \quad \text{.................(1)}$$

Applying KVL on loop II,

$$12 - 5I_1 - 4I_2 - 2.5\,I_2 + 3 = 0$$
$$15 - 5I_1 - 6.5\,I_2 = 0$$
$$5I_1 + 6.5\,I_2 = 15 \quad \text{.................(2)}$$

By solving eqn. (1) and (2)

$$I_1 = 1.375\text{ A}$$
$$I_2 = 1.25\text{ A}$$

Hence, current through 2.5 Ω resistor is 1.25 A

Then, current through 1Ω resistor $= I_1 - I_2$

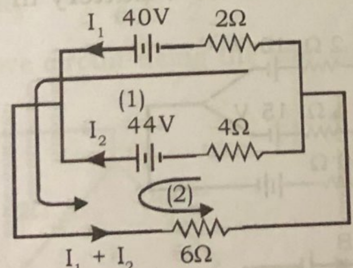
$= 0.125$ A

So, voltage drop across 1Ω resistor $= 0.125 \times 1$

$= 0.125$ V

3. A battery of emf 40 V and internal resistance 2 Ω is connected in parallel with a secondary battery 44V and internal resistance 4Ω. A load resistance of 6Ω is connected across the ends of the parallel circuit. Calculate the current in each battery and the load, by applying Kirchof's laws.

Ans.



$$40 - (I_1 + I_2)6 - 2I_1 = 0$$
$$40 - 6I_1 - 6I_2 - 2I_1 = 0$$
$$8I_1 + 6I_2 = 40 \quad \text{............(1)}$$
$$44 - 6(I_1 + I_2) - 4I_2 = 0$$
$$-6I_1 - 6I_2 - 4I_2 = -44$$

$$6I_1 + 10\,I_2 = 44 \quad \text{...........(2)}$$

By solving eq. (1) & (2)

We get,

$$I_1 = 3.1\ \text{A}$$
$$I_2 = 2.54\ \text{A}$$
$$\& \quad I_1 + I_2 = 5.64\ \text{A}$$

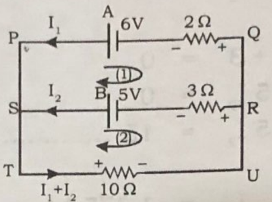
So current supplied by 40 V battery = 3.1 A

current supplied by 44 V battery = 2.54 A

The current passes through 6Ω load = 5.64 A

4. Battery A has an e.m.f. of 6V and an internal resistance of 2Ω. The corresponding vlaues of battery B are 5V and 3Ωrespectively. The two batteries are connected in parallel across a 10Ω resistor. Calculate the current in each branch of the network.

Ans.



Applying KVL to loop PQRSP gives

$$-6 + 2I_1 - 3I_2 + 5 = 0$$
$$2I_1 - 3I_2 = 1 \quad \text{.................(1)}$$

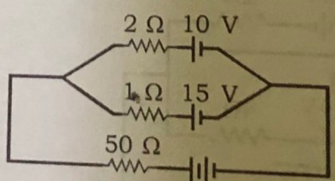
Applying to KVL to loop RSTUR

$$-5 + 3I_2 + 10(I_1 + I_2) = 0$$
$$10\,I_1 + 10\,I_2 + 3I_2 = 5$$
$$10\,I_1 + 13\,I_2 = 5 \quad \text{.................(2)}$$

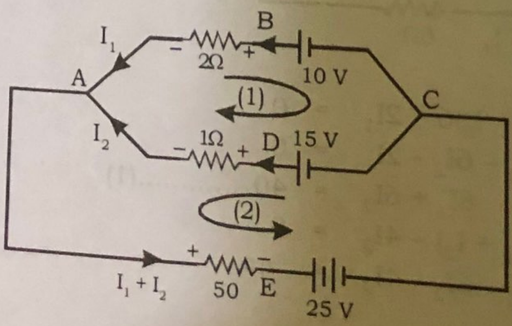
Solving eqn. (1) and eqn. (2) we get

$$I_2 = 0, \; I_1 = 0.5\ \text{A}$$

5. Find the current taken from the 25 V battery in Fig.



Ans.



Applying KVL on loop ABCDA

$$2I_1 - 10 + 15 - I_2 = 0$$

$$2I_1 - I_2 = -5 \quad ....(1)$$

Applying KVL on loop ADCEA

$$I_2 - 15 + 25 + 50\,(I_1 + I_2) = 0$$

By solving eqn. (1) and eqn. (2), we get,

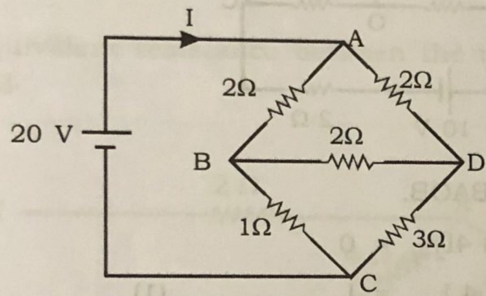
$$I_1 = -1.743 \text{ A} \quad \& \quad I_2 = 1.513 \text{ A}$$

Negative sign of I_1 shows that the current is flowing into 10 V battery and not out of it.

Total current supplied by 25 V source = −1.743 + 1.513 = − 0.23 A

Current of 0.23 A delivered from 25 V source.

6. Find the source current in Fig.

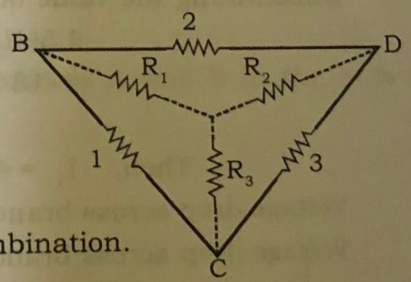


Ans. Convert the delta combination BCD in to equivalent star combination.

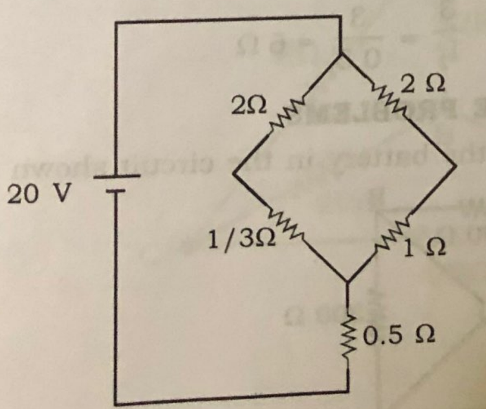
$$\text{Then } R_1 = \frac{R_{BC} \times R_{BD}}{R_{BC} \times R_{BD} + R_{DC}} = \frac{2 \times 1}{6} = \frac{1}{3}\Omega$$

$$\text{Similarly} \quad R_2 = \frac{6}{6} = 1\ \Omega$$

$$R_3 = \frac{3}{6} = \frac{1}{2}\Omega$$



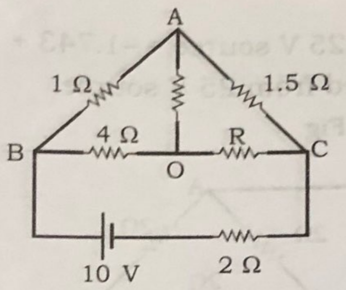
Then re-draw the above circuit using the star combination.



Effective resistance $R_{eq} = 0.5 + 3 \parallel 7/3 = 0.5 + \dfrac{3 \times \frac{7}{3}}{3 + \frac{7}{3}} = 1.8125\ \Omega$

Current supplied by the source $= \dfrac{20}{1.8125} = 11.03\ A$

7. In figure, find the value of R and the current flowing through it when the current is zero in branch OA.



Ans. Applying KVL on loop BAOB.

We get,

$$-I_1 \pm 0 + 4I_2 = 0$$

$$4 I_2 = I_1 \quad \text{..............(1)}$$

Applying KVL on loop BACB.

$$-I_1 - 1.5\, I_1 - 2(I_1 + I_2) + 10 = 0$$

$$-4.5\, I_1 - 2I_2 = -10 \quad \text{..............(2)}$$

Substituting the value of I_1 on equation (2)

$$-4.5(4I_2) - 2I_2 = -10$$

$$-18\, I_2 - 2I_2 = -10$$

$$I_2 = 0.5\ A$$

Then, $I_1 = 4 \times 0.5 = 2\ A$

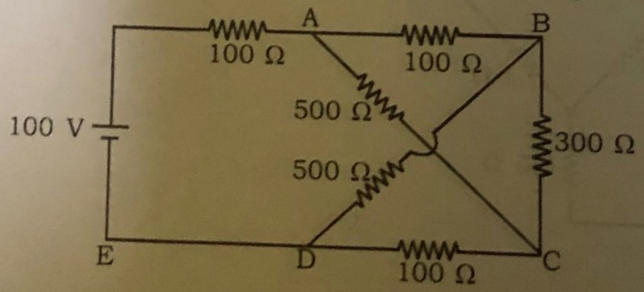
Voltage drop across branch AC $= 2 \times 1.5 = 3\ V$

Voltage drop across branch OC is also 3V.

So, unknown resistance $R = \dfrac{3}{I_2} = \dfrac{3}{0.5} = 6\ \Omega$

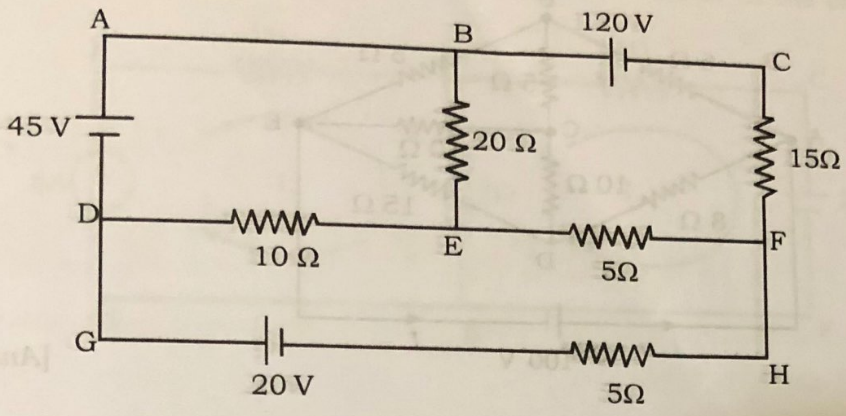
PRACTICE PROBLEMS

1. Determine the current supplied by the battery in the circuit shown in figure.



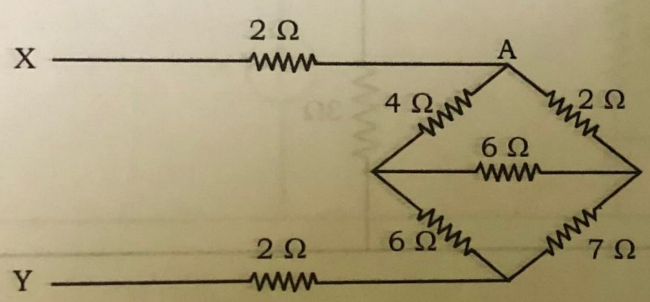
[*Ans.* $\frac{3}{10}$]

2. Determine the current through 20 Ω resistor in the circuit shown in Fig.



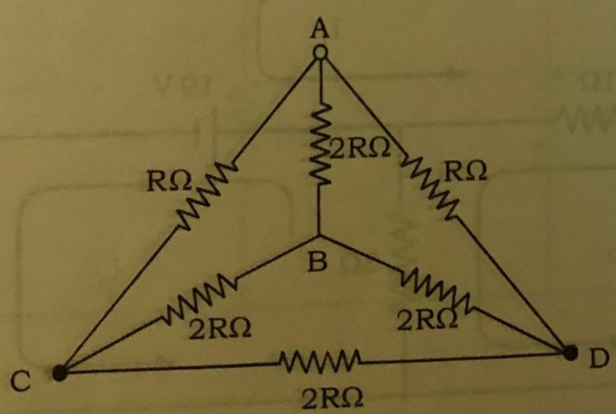
[*Ans.*2.554 A]

3. Find the equivalent resistance between the terminals X and Y in the network shown in Fig.



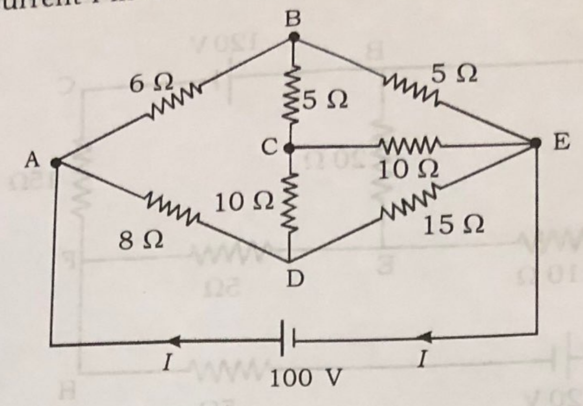
[*Ans.* 8.67 Ω]

4. Find the equivalent resistance between the terminals A and B in the network shown in Fig.



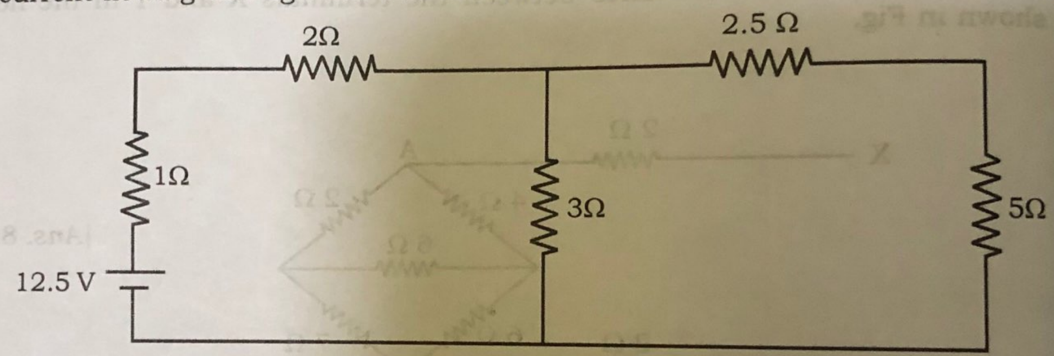
[*Ans.* $\frac{6}{7}$R Ω]

5. Find the current I in the network shown in Fig.



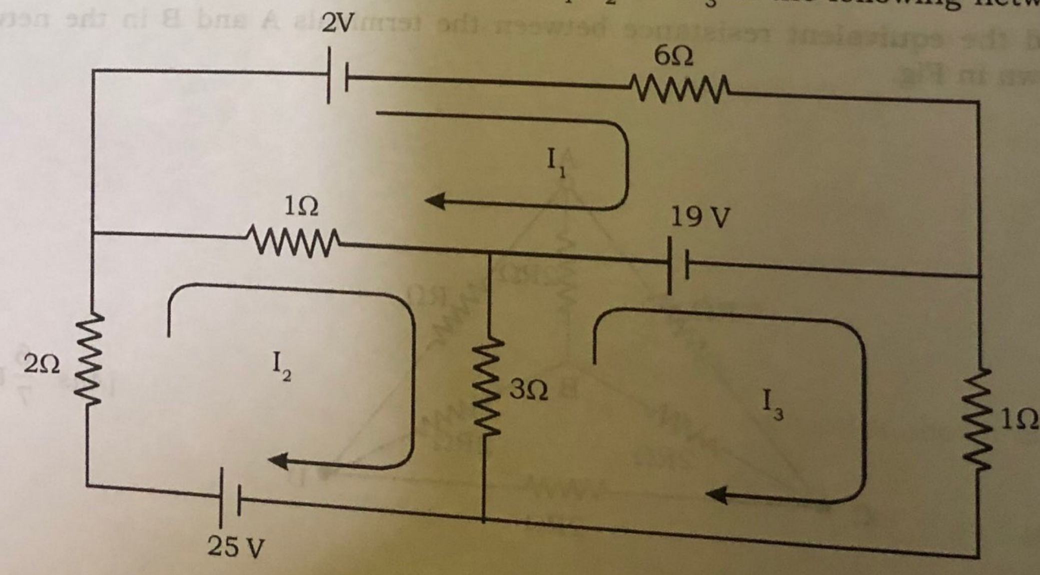
[*Ans.* 15.41 A]

6. With the help of mesh current method, find the magnitude and direction of the current flowing through the 1 Ω resistor in the following network.



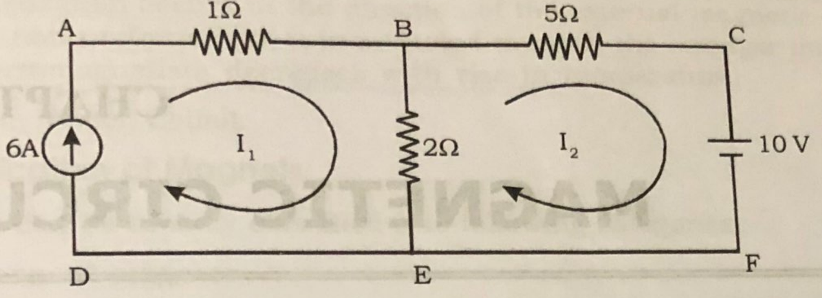
[*Ans.* 2.43 A]

7. By mesh analysis, find mesh currents I_1, I_2 and I_3 in the following network.



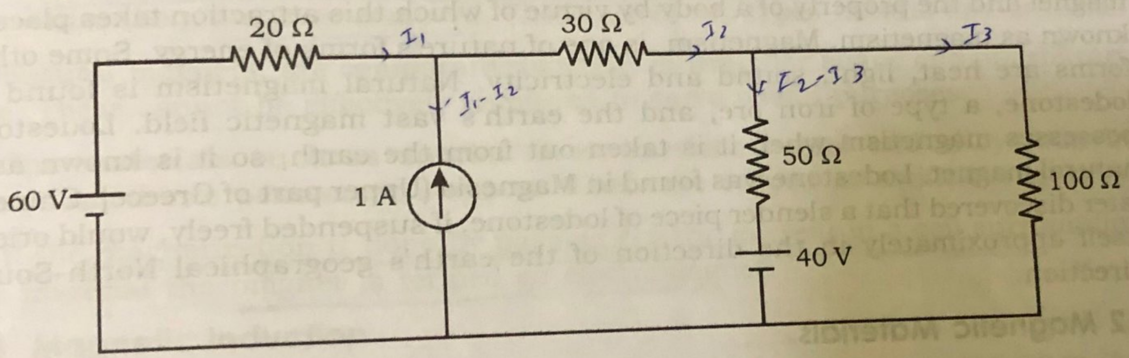
[*Ans.* I_1 = 2.95 A, I_2 = 3.65 A, I_3 = –2.01 A]

8. By mesh analysis, find the current through 2Ω resistor in the circuit.



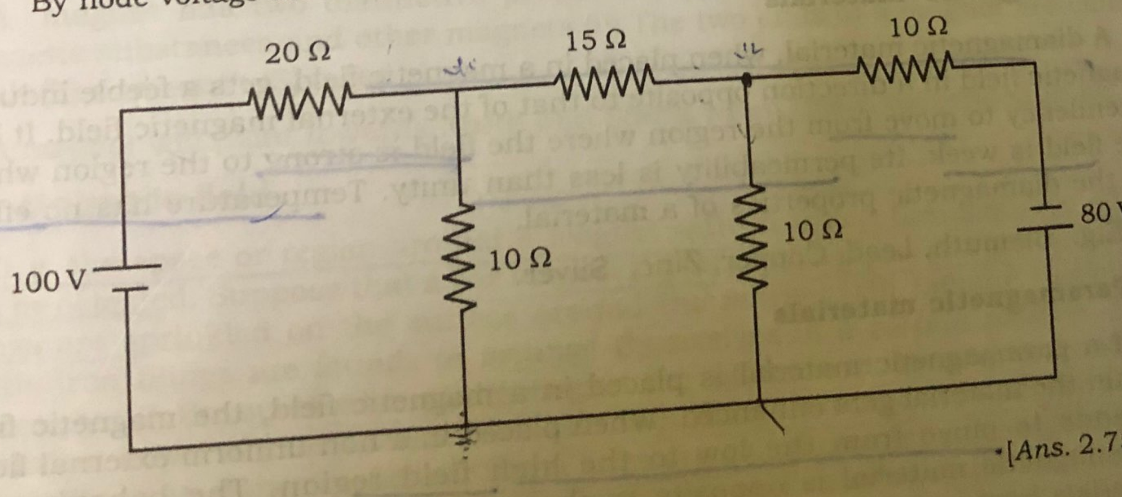
[*Ans.* 5.71 A]

9. By mesh analysis, find the current through 100 Ω resistor in the circuit.



[*Ans.* 0.48 A]

10. By node voltage method, find the current through 15 Ω resistor in the circuit.



[*Ans.* 2.75 A]